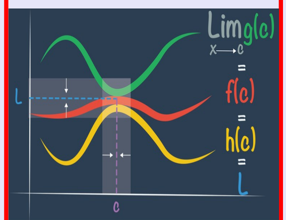
Math 261

Spring 2022

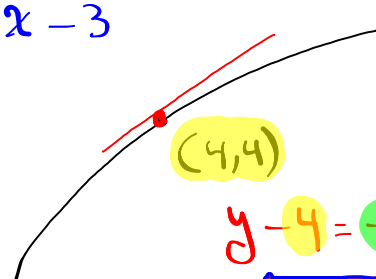
Lecture 11



Class QZ 7

Find equation of the tan. line to the graph of $f(x) = \dfrac{4}{x-3}$ at $x = 4$.

$\boxed{f(4) = \dfrac{4}{4-3} = \dfrac{4}{1} = \boxed{4}}$



$(4,4)$

$f'(x) = \dfrac{0(x-3) - 4(1)}{(x-3)^2}$

$y - 4 = -4(x - 4)$

$f'(x) = \dfrac{-4}{(x-3)^2}$

$m = f'(4) = -4$

$\boxed{y = -4x + 20}$

Find $f'(x)$

1) $f(x) = \frac{1}{4}x^4 - \cos x$

$f'(x) = \frac{1}{4}\cdot 4x^3 - (-\sin x)$ $\quad \boxed{f'(x) = x^3 + \sin x}$

2) $f(x) = 4\sec x + 2\tan x$

$f'(x) = 4\cdot \sec x \tan x + 2\cdot \sec^2 x$

$= \boxed{2\sec x\left[2\tan x + \sec x\right]}$

3) $f(x) = \sec x \cot x$

$f'(x) = \sec x \underbrace{\boxed{\tan x \cdot \cot x}}_{1} + \sec x \cdot (-\csc^2 x)$

$= \sec x - \sec x \csc^2 x = \sec x\left[1 - \csc^2 x\right]$

$= \frac{1}{\cos x}\left[1 - \frac{1}{\sin^2 x}\right]$

$= \frac{1}{\cos x}\left[\frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}\right]$

$= \frac{1}{\cos x}\cdot\frac{\sin^2 x - 1}{\sin^2 x}$

$= \frac{1}{\cancel{\cos x}}\cdot\frac{-\cos^{\cancel{2}} x}{\sin^2 x}$

$= \frac{-\cos x}{\sin^2 x}$

$= -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$

$f(x) = \sec x \cot x$

$f(x) = \frac{1}{\cos x}\cdot\frac{\cos x}{\sin x} = \frac{1}{\sin x}$

$f'(x) = \frac{0\cdot \sin x - 1\cdot \cos x}{\sin^2 x}$

$f'(x) = \frac{-\cos x}{\sin^2 x}$

$\boxed{f'(x) = -\cot x \cdot \csc x}$

$f(x) = \frac{1 - \sec x}{\tan x}$ $\quad 1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$f'(x) = \frac{(0 - \sec x \tan x)\cdot \tan x - (1 - \sec x)\cdot \sec^2 x}{\tan^2 x}$

$= \frac{-\sec x \tan^2 x - \sec^2 x(1 - \sec x)}{\tan^2 x}$

$= \frac{-\sec x\left[\tan^2 x + \sec x(1 - \sec x)\right]}{\tan^2 x}$

$= \frac{-\sec x\left[\cancel{\sec^2 x} - 1 + \sec x - \cancel{\sec^2 x}\right]}{}$

$= \frac{-\sec x\left[\sec x - 1\right]}{\tan^2 x} = \frac{-\sec x\left[\sec x - 1\right]}{\sec^2 x - 1}$

$= \frac{-\sec x\left[\cancel{\sec x - 1}\right]}{(\sec x + 1)\cancel{(\sec x - 1)}} = \boxed{\frac{-\sec x}{\sec x + 1}}$

$f(x)=\dfrac{\tan x-1}{\sec x}$

$f'(x)=\dfrac{(\sec^2 x-0)\cdot\sec x-(\tan x-1)\cdot\sec x\tan x}{\sec^2 x}$

$1+\tan^2 x=\sec^2 x$

$1=\sec^2 x-\tan^2 x$

$=\dfrac{\cancel{\sec x}\left[\boxed{\sec^2 x-\tan^2 x}+\tan x\right]}{\sec^{\cancel{2}} x}$

$\boxed{f'(x)=\dfrac{1+\tan x}{\sec x}}$

$f(x)=\dfrac{\tan x-1}{\sec x}=\dfrac{\frac{\sin x}{\cos x}-1}{\frac{1}{\cos x}}=\dfrac{\sin x-\cos x}{1}$

LCD: $\cos x$

$f(x)=\sin x-\cos x$

$f'(x)=\cos x-(-\sin x)$

$f'(x)=\cos x+\sin x$

$\boxed{y=A\sin x+B\cos x}$

Find and Simplify $y''+y'-2y$

$y''=\frac{d}{dx}[y']$

A & B are constants.

$y'=A\cos x+B(-\sin x)$

$\boxed{y'=A\cos x-B\sin x}$

$\boxed{y''=-A\sin x-B\cos x}$

$y''+y'-2y=-A\sin x-B\cos x+A\cos x-B\sin x-2(A\sin x+B\cos x)$

$=-A\sin x-B\cos x+A\cos x-B\sin x-2A\sin x-2B\cos x$

$=\boxed{(A-3B)\cos x+(-3A-B)\sin x}$

Find A and B such that $y''+y'-2y=\sin x$

$\underbrace{(A-3B)}_{0}\cos x+\underbrace{(-3A-B)}_{1}\sin x=\underbrace{\sin x}_{1}$

Solve $\begin{cases}A-3B=0\rightarrow A=3B\\-3A-B=1\end{cases}$

$-3(3B)-B=1$

$-10B=1$

$\boxed{B=\frac{-1}{10}}$

$A=3\left(\frac{-1}{10}\right)$

$\boxed{A=\frac{-3}{10}}$

$\boxed{\begin{array}{l}A=\frac{-3}{10}\\B=\frac{-1}{10}\end{array}}$

Find $f'(x)$ for $f(x) = \sin 2x$

$f(x) = 2\sin x \cos x$

$f'(x) = 2\left[\cos x \cdot \cos x + \sin x \cdot (-\sin x)\right]$

$= 2\left[\cos^2 x - \sin^2 x\right]$

$\boxed{f'(x) = 2\cos 2x}$

Chain Rule

If $F(x) = (f \circ g)(x) = f(g(x))$

Assuming all derivatives are defined.

$\boxed{F'(x) = f'(g(x)) \cdot g'(x)}$

$y = f(u)$ and $u = g(x)$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

$y = u^2$ and $u = 5x - 1$ $\Rightarrow y = (5x-1)^2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 5 = 10u = 10(5x-1)$$

$y = \sin u \quad , \quad u = x^2 + x \quad \Rightarrow y = \sin(x^2+x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos u \cdot (2x+1) = (2x+1)\cos u$$

$$= \boxed{(2x+1)\cdot\cos(x^2+x)}$$

$y = \sin u \qquad u = 2x \quad \Rightarrow y = \sin 2x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos u \cdot 2 = 2\cos u = \boxed{2\cos 2x}$$

$y = \sqrt{x^2+1}$, find y'.

Let $u = x^2 + 1$

So $y = \sqrt{u} \quad , \quad u = x^2+1$

$y = u^{1/2} \quad , \quad u = x^2+1$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\cancel{2}} u^{\frac{1}{2}-1} \cdot \cancel{2}x$$

$$= u^{-1/2} \cdot x = \frac{x}{u^{1/2}}$$

$$= \frac{x}{\sqrt{u}}$$

$$\boxed{y' = \frac{x}{\sqrt{x^2+1}}}$$

$f(x) = \sin x^3$

$f'(x) = \cos(x^3) \cdot 3x^2$

$\boxed{f'(x) = 3x^2 \cos x^3}$

$f(x) = \sin 2x$

$f'(x) = \cos(2x) \cdot 2$

$\boxed{f'(x) = 2\cos 2x}$

$f(x) = (x^3 - 5x^2)^4$

$f'(x) = 4(x^3 - 5x^2)^{4-1} \cdot (3x^2 - 10x)$

$= \boxed{4(3x^2 - 10x)(x^3 - 5x^2)^3}$

$$f(x) = \frac{1}{\sqrt[3]{x^3+6x-2}}, \quad \text{find } f'(x)$$

$$f(x) = \frac{1}{(x^3+6x-2)^{1/3}} = (x^3+6x-2)^{-1/3}$$

$$f'(x) = \frac{-1}{3}(x^3+6x-2)^{\frac{-1}{3}-1} \cdot (3x^2+6)$$

$$f'(x) = \frac{-1}{\cancel{3}}(x^3+6x-2)^{\frac{-4}{3}} \cdot \cancel{3}(x^2+1)$$

$$= -\frac{x^2+1}{(x^3+6x-2)^{4/3}} = \boxed{\frac{-(x^2+1)}{(x^3+6x-2)\sqrt[3]{x^3+6x-2}}}$$

$$f(x) = \left(\frac{4}{x-3}\right)^5, \quad \text{find } f'(x).$$

$$f'(x) = 5\left(\frac{4}{x-3}\right)^{5-1} \cdot \frac{d}{dx}\left[\frac{4}{x-3}\right]$$

$$= 5\left(\frac{4}{x-3}\right)^4 \cdot \frac{-4}{(x-3)^2} = \frac{-20 \cdot 4^4}{(x-3)^4(x-3)^2}$$

$$= \frac{-20 \cdot 256}{(x-3)^6}$$

$$\boxed{f'(x) = \frac{-5120}{(x-3)^6}}$$

March 16, 2022

$f(x) = \sqrt{\sec x^3}$

Outer function $\rightarrow$ Square-Root Function

Middle " $\rightarrow$ Sec Function

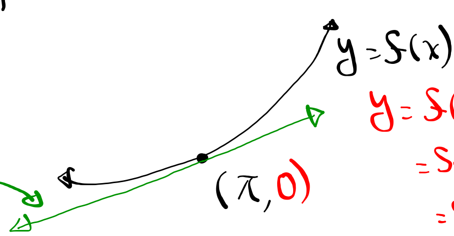
inner Function $\rightarrow$ x^3

$f(x) = \left[\sec x^3\right]^{1/2}$

$f'(x) = \frac{1}{2}\left[\sec x^3\right]^{\frac{1}{2}-1} \cdot \sec x^3 \tan x^3 \cdot 3x^2$

$$\boxed{f'(x) = \frac{3x^2 \sec x^3 \tan x^3}{2\sqrt{\sec x^3}}} \Rightarrow f'(x) = \frac{3x^2 \tan x^3 \sqrt{\sec x^3}}{2}$$

Find equation of the tangent line to the graph of $f(x) = \sin(\sin x)$ at $x = \pi$.



$y = f(\pi)$

$= \sin(\sin\pi)$

$= \sin(0)$

$= 0$

$m = f'(x)\Big|_{(\pi,0)}$

$= \cos(\sin\pi) \cdot \cos\pi$

$= \cos(0) \cdot \cos\pi$

$= 1 \cdot (-1)$

$m = -1$ Slope of tan. line

$f(x) = \sin(\sin x)$

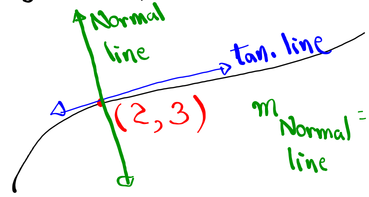
$f'(x) = \cos(\sin x) \cdot \cos x$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - \pi)$

$\boxed{y = -x + \pi}$

Find equation of the normal line to the graph of $f(x) = \sqrt{1+x^3}$ at $x=2$.



$f(2) = 3$ ✓

$$m_{\text{Normal line}} = \frac{-1}{m_{\text{tan. line}}}$$

$$m_{\text{tan. line}} = f'(x)\Big|_{(2,3)}$$

$$f(x) = (1+x^3)^{1/2}$$

$$f'(x) = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2$$

$$f'(x) = \frac{3x^2}{2\sqrt{1+x^3}}$$

$$f'(2) = \frac{3 \cdot 2^2}{2 \cdot \sqrt{1+2^3}} = 2$$

$$m_{\text{tan. line}} = 2$$

$$m_{\text{Normal line}} = \frac{-1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{-1}{2}(x-2)$$

$$\boxed{y =}$$

Class QZ 8

$f(x) = \cos(a^2 + x^3)$ Find $f'(x)$.

No need to Simplify.

$$f'(x) = -\sin(a^2+x^3) \cdot 3x^2$$

$$= \boxed{-3x^2 \sin(a^2+x^3)}$$